5. C. S. Wells, in: Viscous Drag Reduction (edited by C. S. Wells), Plenum Press, New York (1969).
6. V. A. Gorodtsov, Inzh.-Fiz. Zh., 28, No. 6 (1975).
7. W. A. Meyer, AIChE J., 12, No. 3 (1966).
8. V. A. Gorodtsov and V. S. Belokon', Inzh.-Fiz. Zh., 25, No. 6 (1973).
9. L. Landweber and M. Poreh, J. Ship. Res., 17, No. 4 (1973).
10. V. A. Gorodtsov, Inzh.-Fiz. Zh., 28, No. 3 (1975).
11. P. S. Virk et al., Trans. ASME, Ser. E: Appl. Mech., 92, No. 2 (1970).
12. P. S. Granville, J. Hydronautics, 6, No. 1 (1972).

FLUID FRICTION OF A POLYMER SOLUTION FLOWING IN A LARGE-DIAMETER PIPE

Yu. F. Ivanyuta and L. A. Chekalova

UDC 532.135

Experimental data are given from comparative tests to determine the fluid friction in flows of a Polyox solution with a concentration $c = 7 \cdot 10^{-6}$ g/cm^3 in pipes having diameters d = 35.5 mm and d = 514 mm.

§1. The discovery of drag reduction effected in turbulent flows of water near a rigid wall by the addition of small quantities of high-molecular-weight compounds (polymers) to the flow has in the last few years motivated extensive research aimed at explaining this phenomenon and devising practical methods for predicting the attainable net effect. One of the possible techniques for calculating the net effect of drag reduction in pipe flows of polymer solutions has been proposed by the authors [1]. The method is based on universal graphs of the investigated influence of polymer additives as a function of the type of polymer, flow velocity in the pipe, and concentration of the solution [1, 2]. However, all the experimental material used for analysis and plotted in the form of universal graphs refers to flows in pipes whose diameters do not exceed 35 mm and the flow velocity is such that the range of Reynolds numbers is $7 \cdot 10^3$ to $3 \cdot 10^5$. The published data on the influence of polymer additives in a flow on the friction in pipes have also been obtained in the same range of pipe diameters (d < 50 mm) and Reynolds numbers ($Re < 5 \cdot 10^5$) and correspond qualitatively to the results of our earlier generalization [1, 2].

Thus, all the cited experiments have been conducted under conditions of a limited range of Reynolds numbers in comparatively small-diameter pipes. The difficulties inherent in the experimental investigation of the characteristics of turbulent flow of polymer solutions in pipes of large diameter stem primarily from the

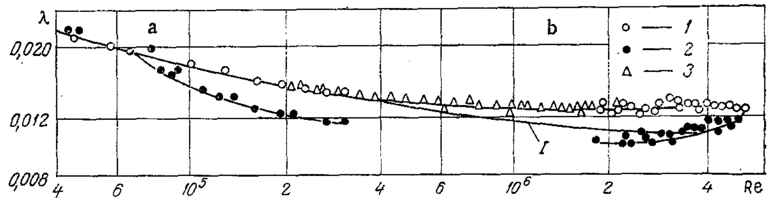

Fig. 1. Coefficient of fluid friction versus Reynolds number Re for flow of water and a Polyox solution with concentration $c = 7 \cdot 10^{-6}$ g/cm^3 in pipes with diameters d = 35.5 mm (a) and d = 514 mm (b). 1) Water; 2) Polyox solution; 3) Galavics' tests with water [4]: I) $1/\sqrt{\lambda} = 2 \log Re\sqrt{\lambda} - 0.8$.

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 31, No. 3, pp. 493-498, September, 1976. Original article submitted June 16, 1975.

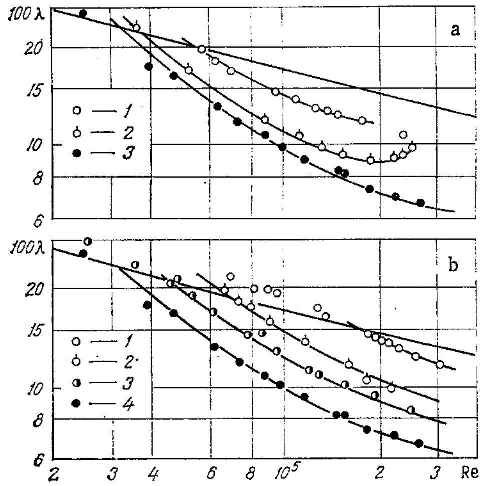

Fig. 2. Coefficient of fluid friction λ versus Reynolds number Re for flows of Polyox solutions at different concentrations in a pipe of diameter d = 35.5 mm. a: 1) $c = 5 \cdot 10^{-6}$ g/cm^3; 2) 10^{-5}; 3) $2 \cdot 10^{-5}$. b: $c = 2 \cdot 10^{-5}$ g/cm^3: 1) fresh solution; 2) solution after one run; 3) after two runs; 4) after three runs.

need to prepare a large quantity of the solution (as much as several thousand tons). This fact accounts for the publication of only one known paper [3], in which a flow of a polyacryl amide (Separan AP-30) solution with a concentration $c = 5 . 10^{-5}$ g/cm^3 in a pipe with diameter d = 254 mm at Reynolds numbers $Re = 7 \cdot 10^4$ to $7 \cdot 10^5$ is investigated.

The results of this investigation can only be viewed as qualitative; they show that a considerable drag reduction is to be expected (40% in the experimental data) in large-diameter pipes at large Reynolds numbers. The investigation, however, does not support any conclusion as to whether this reduction is less than or greater than the reduction in flows of the same solutions in smaller-diameter pipes.

The objective of the present experiments was to compare directly the effects of polymer additives in flows in large- and small-diameter pipes, as well as to assess the possibility of predicting the magnitude of those effects for large Reynolds numbers.

§2. All the experiments were conducted on two different arrangements. The small-diameter pipe (d = 35.5 mm) was connected into a standard closed-cycle water tunnel system with a capacity of ~500 liters [1]. The large-diameter pipe (d = 514 mm) was installed in a water tunnel with open-cycle operation. A pump supplied water from an open tank into the pipe with d = 514 mm, which had a total length of 40 m. The volume of the tank, including the bypass, was ~5000 tons. The straight pipe section containing the measurement part had a length of 25 m.

Data from measurements of the fluid friction associated with water flowing in the closed-cycle hydraulic apparatus are given in Fig. 1, where they evince a fully developed turbulent flow regime in the hydraulically smooth pipe in the range of Reynolds numbers $Re = 10^4$ to $2 \cdot 10^5$.

The fluid friction of water in the pipe with d = 514 mm was measured in the range of Reynolds numbers $Re = 1.8 \cdot 10^6$ to $5.2 \cdot 10^6$ (Fig. 1). These data are in good agreement with the corresponding measurement results obtained by Galavics [4] for water flowing in a technically smooth pipe with a roughness height R/k_s = 1300.

For the experiments we used Polyox WSR-301 (polyethylene oxide), from which a solution having a single concentration was prepared. Polyox powder (40 kg) was dispersed uniformly over the surface of the open part of the tank. Due to the large surface area of the tank it was possible to disperse the powder uniformly without lumping. As soon as the solution was prepared a small pump was immediately turned on, pumping the solution from the open part of the tank into the bypass channel to promote vigorous mixing.

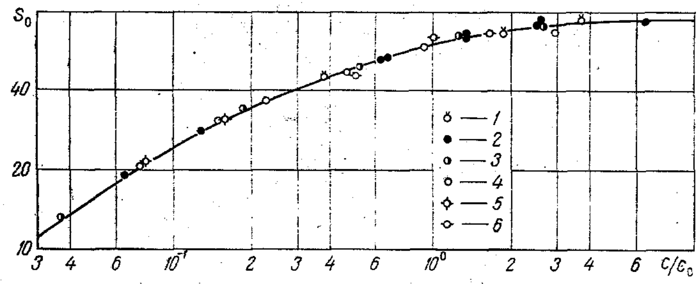

Fig. 3. Maximum frictional drag reduction $S = (\tau_W - \tau_p)/\tau_W$ versus relative concentration c/c_0 of polymer solution at temperature t = 10 to 20°C. 1) Polyox, $c = 2.7 \cdot 10^{-6}$ g/cm³ [1]; 2) Polyox, $c = 7.7 \cdot 10^{-6}$ g/cm³; 3) polyacryl amide, $c = 2.7 \cdot 10^{-5}$ [1]; 4) guar gum, $c = 6.7 \cdot 10^{-4}$ g/cm³ [1]; 5) guar gum, $c = 6.7 \cdot 10^{-4}$ g/cm³ [5]; 6) Polyox, $c = 2 \cdot 10^{-5}$ g/cm³.

Two hours after the instant of preparation the fluid friction of the solution was determined as a function of its flow velocity in the large-diameter pipe. The measurement data are plotted in dimensionless form as λ versus Re in Fig. 1. The experiment was repeated three times and yielded identical results. During each experiment the prepared solution passed only once through the pump and experimental pipe due to the large volume of the tank in which the solution was prepared and due to the brevity of the measurement periods.

Concurrently with the tested flows of Polyox solution in the large-diameter pipe (d = 514 mm) we tested the same solution in the small water-tunnel system with a working section of diameter d = 35.5 mm. The required quantity of the solution (~ 500 liters) was drawn from the open tank and transferred into the test section without the use of a pump. The results obtained for this solution flowing in the pipe with diameter d = 35.5 mm are also given in Fig. 1. Since the Polyox solution was tested simultaneously in pipes with diameters d = 514 mm and d = 35.5 mm, its quality may be presumed uniform.

A comparison of the results of the tests shows that the effectiveness (property of reducing frictional drag) of the same Polyox solution flowing in pipes of different diameters is identical (S_0 = 20%).

It may therefore be concluded on the basis of the given measurements that the maximum drag reduction in pipes is determined solely by the particular concentration of the polymer solution and is independent of the pipe diameter.

APPENDIX

Determination of the Solution Concentration

For preparation of the polymer solution 40 kg of powdered Polyox were dispersed over the surface in the open part of the tank. However, some of the powder found its way into stagnant regions of the tank (in the corners of the open tank and partially in the bypass channel), and some of the powder settled to the bottom. Consequently, not all of the powder became mixed and involved in the experiments. It was therefore difficult to determine the true concentration of the prepared solution. For a more precise determination of the solution concentration and its quality we conducted a preliminary study.

Polymers, as we know [1, 2], can be characterized by two parameters: 1) the threshold value τ_* of the tangential frictional stress; 2) the characteristic concentration c_0 of a solution of the given type of polymer for maximum drag reduction in the flow, S_0 = 60%. The first of these parameters characterizes the condition for existence of the effect when $\tau > \tau_*$, and the second the specific effectiveness of the polymer from the standpoint of drag reduction. The values of these parameters are affected not only by the type of polymer, but also by the conditions under which the solution is prepared. It is well known that the effectiveness of solutions of the same concentration depends on the solution temperature, the time to prepare it, the magnitude of the acting shear stresses at the time of solution preparation, the active duration of those stresses, etc. It is noted that in all cases, except the influence of temperature, simultaneous variation of both parameters is observed.

It was assumed in the present study that for one type of polymer at a constant solution temperature it should be possible to find a relationship between the variations of the two parameters; accordingly, we

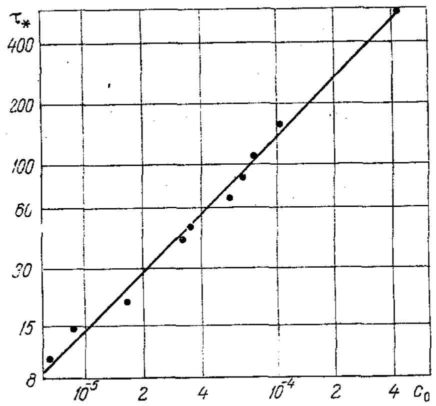

Fig. 4. Threshold frictional stress τ_*, dyn/cm^2, versus concentration c_0, g/cm^3, $t \simeq 10°C$.

conducted a series of tests in the closed-cycle water tunnel system with a working section of diameter d = 35.5 mm, using Polyox solutions from the same batch as that from which the 40 kg were taken for preparation of the polymer solution in the main experiment. The solutions were prepared in concentrations of $5 \cdot 10^{-6}$, 10^{-5}, and $2 \cdot 10^{-5}$ g/cm^3. The measured values of the fluid friction of the solutions as a function of their flow velocity in the pipe with diameter d = 35.5 mm are given in dimensionless plots of λ versus Re in Fig. 2a, b. The solution with a concentration of $2 \cdot 10^{-5}$ g/cm^3 was tested repeatedly, providing a basis for estimation of the decrease in its effectiveness from one run to another (Fig. 2b). The temperature of the solutions fluctuated only slightly (7 to 12°C), corresponding to the temperature of the main experiment.

The results of the foregoing measurements were used to determine the initial threshold Reynolds numbers, the corresponding threshold tangential frictional stresses τ_* at the wall, and the maximum drag reduction S_0 attained in each specific case. The drag reduction S_0 corresponds to the parameter c_0 characterizing the effectiveness of the given solution as determined from Fig. 3, which is taken in part from our earlier work [1].

For example, in the case of a Polyox solution with $c = 2 \cdot 10^{-5}$ g/cm^3 (Fig. 2) we obtain as a result of a second run in the pipe with diameter d = 35.5 mm:

$$\mathrm{Re_{thr}} = 4.5 \cdot 10^4; \qquad v_{\mathrm{thr}} = 1.72\ \mathrm{m/sec}; \qquad \lambda_{\mathrm{thr}} = 0.022;$$

$S_0 = 41\%$; $c/c_0 = 0.3$ (on the basis of the dependence given in Fig. 3).

For these conditions the value of the first parameter τ_* (threshold frictional stress) is

$$\tau_* = \frac{\lambda_{\mathrm{thr}}}{8} \rho v_{\mathrm{thr}}^2 \cong 81\ \mathrm{dyn/cm^2},$$

and the value of the second parameter c_0 (characteristic concentration of solution) is

$$c_0 = c/(c/c_0)_{S_0=41\%} = 2 \cdot 10^{-5}\,\mathrm{g/cm^3}/0.3 = 6.7 \cdot 10^{-5}\ \mathrm{g/cm^3}.$$

The data obtained and processed as indicated for pipe flows of Polyox solutions yield a simple relationship between the two parameters (Fig. 4). Consequently, for one given type of polymer it is possible to estimate the effectiveness of that solution even in the event of its degradation.

The curves given in Figs. 3 and 4 also permit us to determine the concentration of the solution when the tangential frictional stress τ_* at the wall and the maximum drag reduction S_0 at the wall are known, as in experiments with Polyox solutions flowing in small- and large-diameter pipes. For example, the threshold tangential frictional stress at the wall in these experiments was (Fig. 1) $\tau_* = 130$ dyn/cm^2.

For this value of the first parameter, according to the curve in Fig. 4, it is possible to determine the value of the second parameter: $c_0 = 10^{-4}$ g/cm^3.

We use the curve of Fig. 3 to find the dimensionless solution concentration corresponding to a value of the drag reduction $S_0 = 20\%$, namely $c/c_0 = 0.07$.

Consequently, the true value of the concentration of the Polyox solution in the main experiments, based on our determination of the effective drag reduction in flow of the solution in pipes of different diameters, is

$$c = c_0 (c/c_0)_{S_0=20\%} = 10^{-4}\cdot 0.07 = 7\cdot 10^{-6}\ \ \mathrm{g/cm^3}.$$

This result does not conflict with the expected concentration of the Polyox solution ($c < 8\cdot 10^{-6}$ g/cm^3) when the inevitable material losses associated with the particular technique for preparation of the solution are taken into account.

NOTATION

d, pipe diameter; t, flow temperature; c, weight concentration of polymer solution; ν, kinematic viscosity of water; ν_p, kinematic viscosity of polymer solution; $\eta = \nu_p/\nu$, relative viscosity of polymer solution; ρ, density of water; τ_W, τ_p, tangential frictional stresses at the wall in pipe flows of water and polymer solution, respectively; τ_*, threshold tangential frictional stress at the wall; v_S, average velocity in terms of mass flow of liquid in the pipe; λ_W, λ_p, coefficients of fluid friction in pipe flows of water and polymer solution, respectively; Re, Reynolds number; c_0, weight concentration of polymer solution at t, °C, for 60% drag reduction; $S = (\tau_W - \tau_p)/\tau_W$, drag reduction at v_S = const for flow of polymer solution; S_0, maximum drag reduction for flow of a polymer solution of concentration c; c_0, characteristic concentration of polymer solution for maximum drag reduction $S_0 = 60\%$.

LITERATURE CITED

1. Yu. F. Ivanyuta and L. A. Chekalova, Inzh.-Fiz. Zh., 21, No. 1 (1971).
2. Yu. F. Ivanyuta and L. A. Chekalova, Inzh.-Fiz. Zh., 26, No. 5 (1974).
3. R. H. Forester, R. E. Larson, J. W. Haylen, and J. M. Wetzel, J. Hydronautics, 3, No. 1 (1969).
4. H. Schlichting, Boundary-Layer Theory, 6th ed., McGraw-Hill, New York (1968).
5. V. N. Kalashnikov et al., Zh. Prikl. Mekh. Tekh. Fiz., No. 6 (1968).

PERISTALTIC FLOW OF A NON-NEWTONIAN VISCOPLASTIC LIQUID IN A SLOT CHANNEL

V. I. Vishnyakov, K. B. Pavlov, and A. S. Romanov

UDC 532.54: 532.135

The "narrow-band" asymptotic method [5] has been used to consider the peristaltic flow of a viscoplastic medium in a slot channel. It is found that the mode of flow differs substantially from that in a channel with rigid walls when the axial pressure gradient is small.

Considerable attention has recently been given to the flow of liquids in channels with elastic walls in connection with many aspects of biomechanics [1], with particular interest attaching to non-Newtonian fluids with anomalous mechanical properties [2]. One class of non-Newtonian liquid is that of nonlinear-viscosity media, for which the simplest rheological law is one that relates the stress tensor deviator s_{ij} to the strain-rate tensor f_{ij}. In particular, a viscoplastic liquid is a medium with nonlinear viscosity, for which the rheological law can be put in the following form [3]:

$$s_{ij} = 2\left[\eta + \tau_0/(2f_{ij}f_{ij})^{1/2}\right] f_{ij} \quad \text{for} \quad (2s_{ij}s_{ij})^{1/2} \geqslant \tau_0, \qquad f_{ij} = 0 \quad \text{for} \quad (2s_{ij}s_{ij})^{1/2} \leqslant \tau_0. \tag{1}$$

Here we consider the peristaltic motion of a viscoplastic liquid (1) in a slot channel with elastic walls; in the general case, the peristaltic flow of the medium is due to the joint action of the deformable walls and a

N. É. Bauman Moscow Higher Technical School, Moscow. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 31, No. 3, pp. 499-505, September, 1976. Original article submitted June 30, 1975.